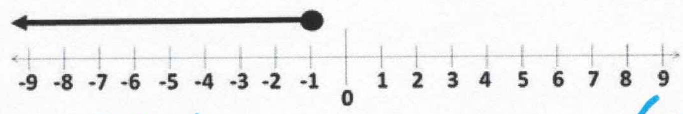
Mr. Michael T. Davis Pre-Calculus	Final Exam Review (Practice Exam) Units 1 through 5 May 16, 2016

Name: Answers

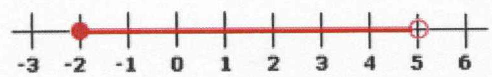
Important Note: This review packet does not contain questions from our current and final unit, namely Unit 5. I will provide review materials for Unit 5 at the end of next week.

1. Write the domain of the graph in both inequality and interval notation.



Inequality: $x \le -1$ Interval: $(-\infty, -1]$

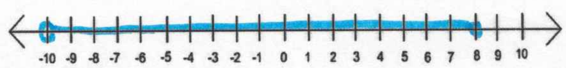
2. Write the domain of the graph in both inequality and interval notation.



Inequality: $-2 \le x < 5$ Interval: $[-2, +5)$

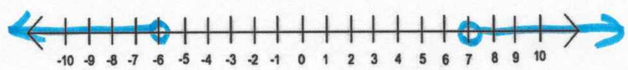
3. Solve each inequality. Graph the solution set on a number line, AND write the solution in interval notation.

a. $-1 < \frac{1}{2}p + 4 \le 8$



$-5 < \frac{1}{2}p \le 4$ $-10 < p \le 8$ $(-10, 8]$

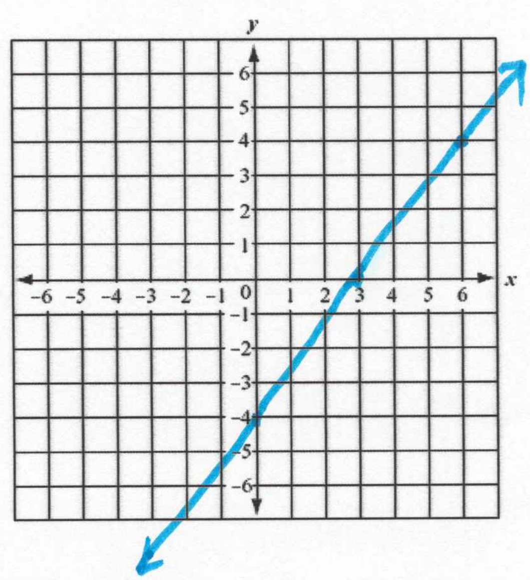
b. $4 - 2x > 16$ *or* $3x - 10 > 11$



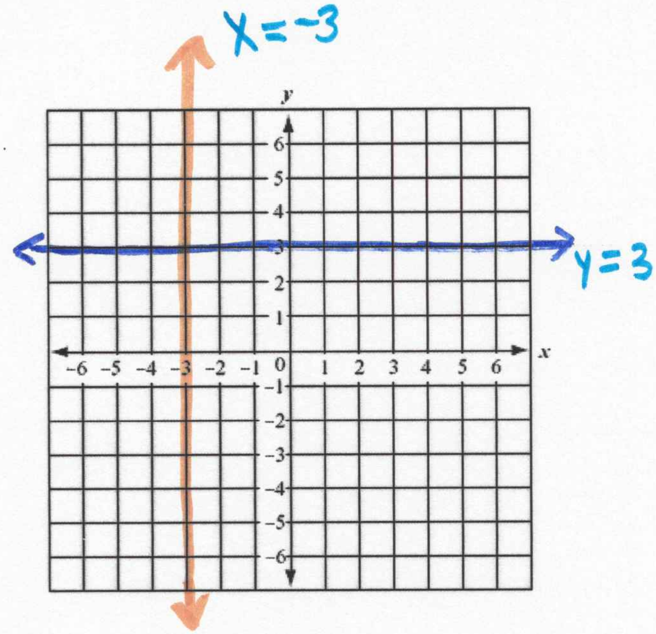
$-2x > 12$ or $3x > 21$
$x < -6$ or $x > 7$

4. Graph the line with equation $4x - 3y = 12$

$(0, -4)$ $(3, 0)$



5. Write an equation for each line shown.



6. Given a line containing the points $(-2,5)$ and $(-4,2)$. Find an equation of the line in slope-intercept form. Convert your equation to standard from.

$m = \frac{2-5}{-4-^-2} = \frac{-3}{-2} = \frac{3}{2}$

$y = \frac{3}{2}x + b$

$5 = \frac{3}{2}(-2) + b$

$5 = -3 + b$

$8 = b$

$y = \frac{3}{2}x + 8$

$2y = 3x + 16$

$2y - 3x = 16$

$3x - 2y = -16$

7. Simplify $49^{\frac{1}{2}}$ $= 7$

8. Simplify $27^{\frac{2}{3}} =$ $\left(27^{1/3}\right)^2 = 3^2 = 9$

9. $\sqrt[4]{16} =$ 2

10. $\frac{35x^4y^8}{7x^{-3}y^5} =$ $= 5x^7y^3$

11. $8x^2y^6 + x^2y^6 =$ $9x^2y^6$

12. $\left(4xy^5\right)^3 =$ $64x^3y^{15}$

13. $\left(8x^2y^6\right)\left(x^2y^6\right) =$ $8x^4y^{12}$

14. Write $\sqrt[4]{y^3}$ in exponential form $y^{3/4}$

15. Square the binomial and simplify $(x-7)^2$ $= (x-7)(x-7) = x^2 - 14x + 49$

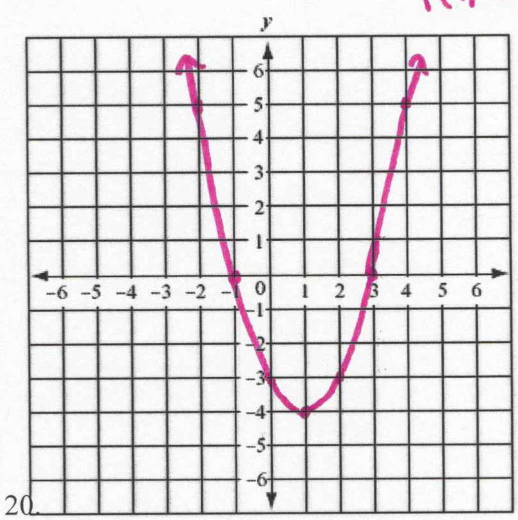
16. Multiply and simplify $(3x-1)(4x+5)$ $= 12x^2 + 15x - 4x - 5 = 12x^2 + 11x - 5$

17. Solve $(x-5)(3x+2)=0$ $x-5=0$ $x=5$ $3x+2=0$ $x=\frac{-2}{3}$ $\{5, \frac{-2}{3}\}$

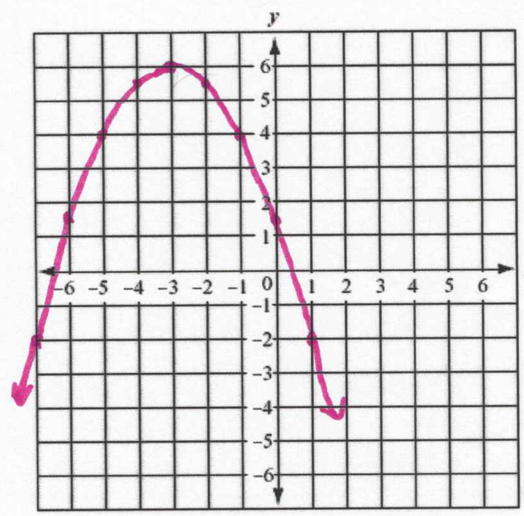
18. Solve $x^2+7x=0$ $x(x+7)=0$ $\{0,-7\}$

19. Solve $x^2-16=0$ $(x-4)(x+4)=0$ $\{4,-4\}$

20. Graph $f(x)=(x+1)(x-3)$ axis $x=1$ $f(1)=-4$



18. Graph $f(x)=-\frac{1}{2}(x+3)^2+6$ $V(-3,6)$



19. Given the parent function $f(x)=x^2$, circle the letter of the correct description of the transformation $T(x)=(x-3)^2+1$.

a. Vertical shift up 3 units and horizontal shift right 1 unit

b. Horizontal shift right 3 units and vertical shift up 1 unit

c. Horizontal stretch by a factor of 3 and vertical shift down 1 unit

d. Horizontal compression by a factor of 3 units and vertical shift down 1 unit

20. Line A has equation $y = \frac{2}{5}x - 6$. Line B contains the point $(4, -9)$ and is perpendicular to line A. Determine an equation in any form for line B.

line B

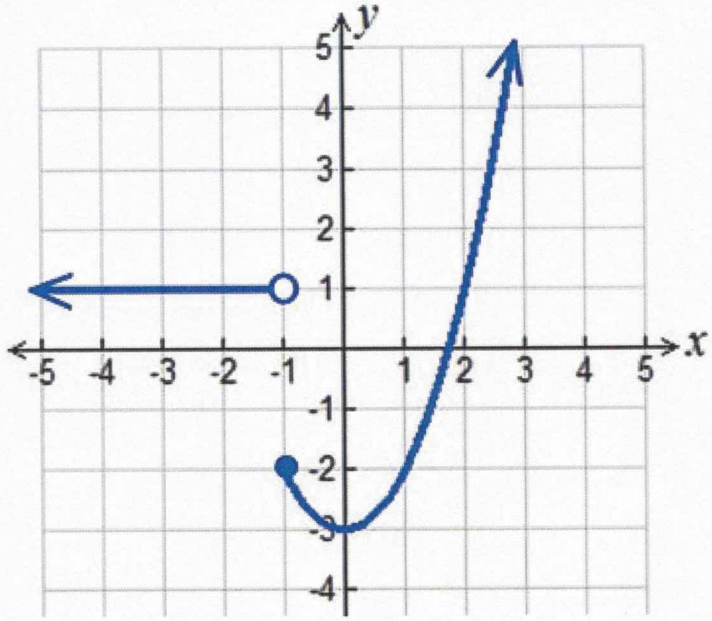
$y = \frac{-5}{2}x + b$

$-9 = \frac{-5}{2}(4) + b$

$-9 = -10 + b$

$1 = b$

$y = -\frac{5}{2}x + 1$



21. Given the graph of $y = f(x)$ above, determine the following (approximate if necessary):

a. Write the domain of $y = f(x)$ in interval notation. $(-\infty, \infty)$

b. Write the range of $y = f(x)$ in interval notation. $[-3, \infty)$

c. Determine the value of $f(0) = -3$

d. Determine the value of $f(-1) = -2$

e. Determine the value(s) of x for which $f(x) = -3$ $\quad x = 0$

f. Using interval notation, determine the values of x for which $f(x)$ is increasing

$(0, \infty)$

g. Using interval notation, determine (estimate) the values of x for which $f(x) < 0$

$[-1, 1.8)$

4.

22. First, convert $f(x) = 3x^2 - 4x - 15$ to factored form, then determine the coordinates of the x-intercepts of the parabola with this equation.

x-intercepts

Let $f(x)=0$

$0 = (3x+5)(x-3)$ $\quad 3x+5=0 \quad x = \frac{-5}{3}$ $\quad x-3=0 \quad x=3$ $\quad \left\{\frac{-5}{3}, 3\right\}$ $\quad (3,0) \quad \left(\frac{-5}{3}, 0\right)$

23. Given functions $f(x) = x^2 - 7$ and $g(x) = x - 3$, evaluate $f(g(x))$

$f(g(x)) = f(x-3) = (x-3)^2 - 7 = x^2 - 6x + 9 - 7 = x^2 - 6x + 2$

24. Factor $x^2 - 6x - 40$

$x^2 - 6x - 40 = (x-10)(x+4)$

25. Factor $x^3 - 7x^2 - \cancel{16}4x + 28$ using grouping $\quad x^3 - 7x^2 - 4x + 28 = 0$

For #'s 27-32, solve for real and imaginary solutions, give all irrational solutions in simplified radical form, and give all imaginary numbers in i-form.

26. Solve $x^2 + 100 = 0$ $\quad x^2 = -100 \quad \sqrt{x^2} = \pm\sqrt{-100} \quad x = \pm i\sqrt{100}$

$x = \pm i \cdot 10 \quad x = \pm 10i \quad \{10i, -10i\}$

27. Solve $2y^3 - 24y = 0$ using factoring (first factor out the common factor)

$2y(y^2 - 12) = 0 \quad 2y = 0 \quad y = 0 \quad y^2 - 12 = 0 \quad y^2 = 12 \quad y = \pm\sqrt{12} \quad \{0, 2\sqrt{3}, -2\sqrt{3}\}$

28. Solve $4x^2 - 25 = 0$ using factoring

$(2x-5)(2x+5) = 0 \quad 2x-5=0 \quad x = \frac{5}{2} \quad 2x+5=0 \quad x = \frac{-5}{2} \quad \left\{\frac{5}{2}, \frac{-5}{2}\right\}$

29. Solve $3x^2 - 4x - 20 = 0$ using factoring

$(3x-10)(x+2) = 0 \quad 3x-10=0 \quad x = \frac{10}{3} \quad x+2=0 \quad x=-2 \quad \left\{\frac{10}{3}, -2\right\}$

30. Solve $2y^2 - 20y + 50 = 0$ using factoring (first factor out the common factor)

$2(y^2 - 10y + 25) = 0 \quad 2(y-5)^2 = 0 \quad y-5=0 \quad y=5 \quad \{5\}$

Double root

31. Solve $x^2 - 4x + 13 = 0$ using the quadratic formula

$x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)} = \dfrac{4 \pm \sqrt{16-52}}{2} = \dfrac{4 \pm \sqrt{-36}}{2} = \dfrac{4 \pm i \cdot 6}{2} = 2 \pm 3i$

32. Simplify $\dfrac{w+1}{16} + \dfrac{w-2}{20} = \dfrac{5(w+1)}{5 \cdot 16} + \dfrac{4(w-2)}{4 \cdot 20} = \dfrac{9w-3}{80}$ $\quad \{2+3i, 2-3i\}$

33. Simplify $\dfrac{2x}{x-5} - \dfrac{3}{x} = \dfrac{x(2x)}{x(x-5)} - \dfrac{(x-5)(3)}{(x-5)(x)} = \dfrac{2x^2 - (3x-15)}{x(x-5)} = \dfrac{2x^2 - 3x + 15}{x(x-5)}$

34. Simplify $\dfrac{n+1}{5mn^2} + \dfrac{m-2}{9m^2n} = \dfrac{9m(n+1)}{9m(5mn^2)} + \dfrac{5n(m-2)}{5n(9m^2n)} = \dfrac{9mn + 9m + 5nm - 10n}{45m^2n^2}$

$= \dfrac{14mn + 9m - 10n}{45m^2n^2}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$(x+2)(x+3)\left(\frac{10}{\cancel{x+2}}\right) = (x+2)\cancel{(x+3)}\left(\frac{15}{\cancel{x+3}}\right)$

$0 = 5x$

$0 = x$

$\{0\}$

$10x + 30 = 15x + 30$

$10x = 15x$

35. Solve $\frac{10}{x+2} = \frac{15}{x+3}$ by multiplying the LCD on both sides

36. Solve $\frac{11}{6y} - \frac{1}{y} = \frac{5}{12}$ by multiplying the LCD on both sides

$12y\left(\frac{11}{6y}\right) - 12y\left(\frac{1}{y}\right) = 12y\left(\frac{5}{12}\right)$ $22 - 12 = 5y$ $10 = 5y$ $2 = y$

$\{2\}$

37. Solve $\frac{22}{x^2} - 1 = \frac{9}{x}$ by multiplying the LCD on both sides

$x^2\left(\frac{22}{x^2}\right) - x^2(1) = x^2\left(\frac{9}{x}\right)$ $22 - x^2 = 9x$ $0 = x^2 + 9x - 22$

$0 = (x+11)(x-2)$ $\{-11, 2\}$

38. The rational function $f(x) = \frac{x^2 - 9}{x^2 - 4x - 21}$ has the following domain:

$x^2 - 4x - 21$

$(x-7)(x+3)$

a. $D:\mathbb{R}, x \neq 3, x \neq -3$ b. $D:\mathbb{R}, x \neq -7, x \neq 3$ c. $D:\mathbb{R}, x \neq 7, x \neq -3$

d. $D:\mathbb{R}, x \neq -4, x \neq -21$ e. $D:\mathbb{R}$ f.

39. The rational function $f(x) = \frac{x^2 + 5x + 6}{x^2 + 1}$ has the following domain:

a. $D:\mathbb{R}, x \neq 3, x \neq 2$ b. $D:\mathbb{R}, x \neq -3, x \neq -2$ c. $D:\mathbb{R}, x \neq -1$

d. $D:\mathbb{R}, x \neq 1, x \neq -1$ e. $D:\mathbb{R}$ f.

40. The rational function $f(x) = \frac{x-8}{x^2 - 25}$ has an x-intercept at:

x-intercept $(x, 0)$

$y = 0$ $f(x) = 0$

$0 = \frac{x-8}{x^2 - 25}$

$0 = x - 8$

a. $y = 0$ b. $x = 5$ c. $x = 8$

d. $y = \frac{8}{25}$ e. There is no x-intercept f.

41. The rational function $f(x) = \frac{-4}{x^2 - 9}$ has a vertical asymptote with equation:

$x^2 - 9$

$(x-3)(x+3)$

a. $y = -4$ b. $x = 9$ c. $x = 3$

d. $x = -3$ & $x = 3$ e. No vertical asymptotes f.

42. The rational function $f(x) = \frac{10}{x^2 - 4}$ has a horizontal asymptote with equation:

The degree of the denominator is greater than the degree of the numerator

a. $y = 0$ b. $x = 2$ c. $y = 10$

d. $x = -2$ & $x = 2$ e. There is no horizontal asymptote f.

43. The rational function $f(x) = \frac{6x^2 + 3x - 1}{2x^2 - 18}$ has a horizontal asymptote with equation:

a. $y = 6$ b. $x = 3$ c. $y = 0$

d. $y = 3$ e. None of the above f.

degrees of numerator and denominator are both 2. The hor. asymptote results from dividing the coefficient of top x^2 by that of bottom.

$y = \frac{6}{2}$ $y = 3$

$f(x) = \frac{(x-4)(x-4)}{(x-4)(x+4)}$ (since $x-4$ is a common factor to the numerator and denominator, the function's graph has a hole at $x=4$

44. The rational function $f(x) = \dfrac{x^2-8x+16}{(x-4)(x+4)}$ has a hole at:

a. $y=1$ b. $y=0$ c. $x=-4$

d. $x=4$ e. There is no hole f.

$x=0$ $f(0) = \frac{0-8}{0^2-25} = \frac{8}{25}$

45. The rational function $f(x) = \dfrac{x-8}{x^2-25}$ has a y-intercept at:

a. $y=8$ b. $x=5$ c. $x=8$

d. $y = \dfrac{8}{25}$ e. There is no y-intercept f.

$f(x) = \frac{x(x-6)}{x(x-2)}$

46. The rational function $f(x) = \dfrac{x^2-6x}{x^2-2x}$ has an x-intercept at:

a. $y=6$ b. $x=3$ c. $x=6$ only

$x \neq 0$

d. $x=0$ & $x=6$ e. There is no x-intercept f.

$f(x)=0$ $0 = \frac{x(x-6)}{x(x-2)}$

$x=6$

47. The rational function $f(x) = \dfrac{x+5}{x^2+7x+10}$ has a vertical asymptote with equation:

a. $x=2$ & $x=5$ b. $x=-5$ only c $x=-2$ & $x=-5$

d. $x=-2$ only e. There is no vertical asymptote f.

$f(x) = \frac{x+5}{(x+2)(x+5)}$

48. The rational function $f(x) = \dfrac{-5}{x^2+9}$ has a vertical asymptote with equation:

x^2+9 cannot $=0$ with real x values

a. $y=-3$ & $y=3$ b. $y=-5$ c. $x=-5$

d. $x=-3$ & $x=3$ e. There is no vertical asymptote f.

$x^2+9 \neq 0$

49. Without a calculator evaluate $\log 1000 =$ 3

50. Without a calculator evaluate $\log_5 1 =$ 0

51. Without a calculator evaluate $\log_8 0 =$ undefined (we cannot take the log of zero)

52. Without a calculator evaluate $\log_3 \dfrac{1}{9} =$ -2

53. Without a calculator evaluate $\log_2 32 =$ 5

54. Without a calculator evaluate $\log(-100) =$ undefined (we cannot take the log of negative numbers)

55. Without a calculator evaluate $\log_9 27 =$ $\frac{3}{2}$ $9^x = 27$ $3^{2x} = 3^3$ $2x = 3$ $x = \frac{3}{2}$

56. Expand $\ln(x \cdot y^2) =$ $\ln x + \ln y^2 = \ln x + 2\ln y$

57. Expand $\log\left(\frac{m}{n^3}\right) =$ $\log m - \log n^3 = \log m - 3\log n$

58. Evaluate by condensing $\log 25 + \frac{1}{2}\log 16 =$ $\log 25 + \log 16^{\frac{1}{2}} = \log 25 + \log 4 = \log 100$
$= 2$

59. Evaluate by condensing $\ln 3 + \frac{1}{2}\ln 49 =$ $\ln 3 + \ln 49^{\frac{1}{2}} = \ln 3 + \ln 7 = \ln 21$

60. Without a calculator evaluate $\ln(e^4) =$ 4

61. Without a calculator evaluate $\log(10^5) =$ 5

62. With a calculator evaluate $\log_4 12 =$ $\frac{\log 12}{\log 4} =$

63. Solve for x: $\log(x + 200) = 3$ $10^3 = x + 200$ $1000 = x + 200$ $800 = x$
$\{800\}$

64. Solve for x: $\log_x 64 = 2$ $x^2 = 64$ $x = 8$ $x \neq -8$
the base cannot be negative

65. Solve for x: $\ln x = 2$ $e^2 = x$

66. Solve for y in terms of x: $\log y = 3\log x$

$\log y = \log x^3 \quad y = x^3$

67. Solve for y in terms of x: $\log_x y = 2$

$x^2 = y \quad y = x^2$

68. Without a calculator solve for x: $\log_2(x-6) + \log_2 x = 4$

$\log_2(x^2 - 6x) = 4 \quad 2^4 = x^2 - 6x \quad 0 = x^2 - 6x - 16$

$0 = (x-8)(x+2)$

$x = 8$ only

69. Without a calculator solve for x: $\log_5(x+10) - \log_5 3 = 2$

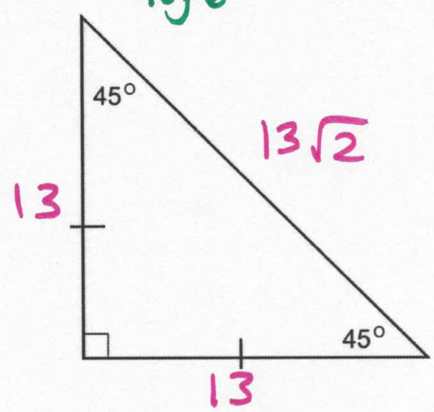
$\log_5 \frac{x+10}{3} = 2 \quad 5^2 = \frac{x+10}{3} \quad 75 = x+10$

$65 = x$

70. Without a calculator solve $6^x = 12$

$\log 6^x = \log 12 \quad x\log 6 = \log 12 \quad x = \frac{\log 12}{\log 6}$

71. Given the special triangle shown with $13\sqrt{2}$ cm as its hypotenuse length, find the lengths of the other two sides.



72. Evaluate $\cos B = \frac{a}{c}$

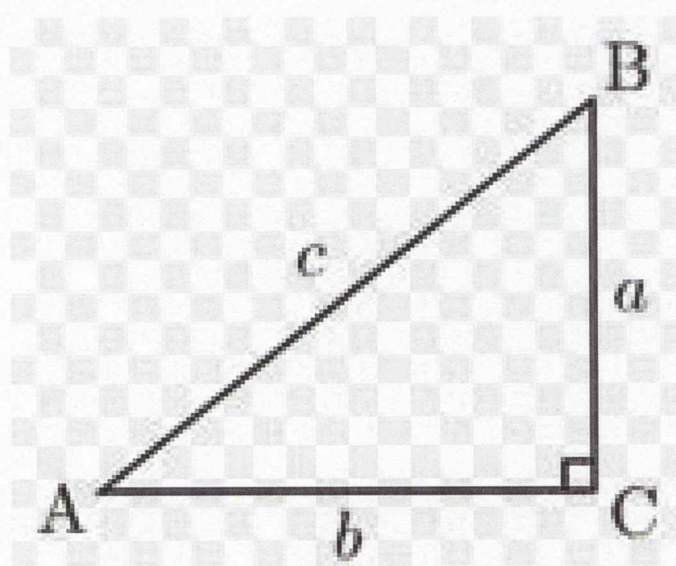
73. Evaluate $\tan A = \frac{a}{b}$

74. Evaluate $\csc B = \frac{c}{b}$

75. Evaluate $\sin A = \frac{a}{c}$

76. Evaluate $\sec A = \frac{c}{b}$

77. Evaluate $\cot B = \frac{a}{b}$



78. Complete the statement (fill in the blank): $\sin \underline{52^\circ} = \cos 38^\circ$

79. Convert $\dfrac{\pi}{6}$ to an angle measure in degrees. 30°

80. Convert 45° to an angle measure in radians, $\dfrac{\pi}{4}$

81. Convert $\dfrac{\pi}{2}$ to an angle measure in degrees. 90°

82. Convert 150° to an angle measure in radians. $\dfrac{5\pi}{6}$

83. Convert $\dfrac{4\pi}{3}$ to an angle measure in degrees. 240°

84. Convert 315° to an angle measure in radians. $\dfrac{7\pi}{4}$

85. Convert $\dfrac{\pi}{15}$ to an angle measure in degrees. 12°

86. Convert 140° to an angle measure in radians. $\dfrac{7}{9}\pi$

87. Write $\sin\dfrac{7\pi}{4}$ in terms of a reference angle (answer with radians)

$-\sin\dfrac{\pi}{4}$

88. Write $\csc 150^\circ$ in terms of a reference angle (answer with degrees)

$\csc 30^\circ$

89. Write $\sec 240^\circ$ in terms of a reference angle (answer with degrees)

$-\sec 60^\circ$

90. Write $\tan\dfrac{2\pi}{3}$ in terms of a reference angle (answer with radians)

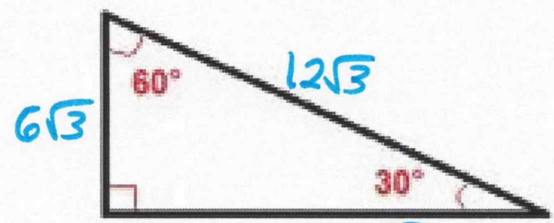
$-\tan\dfrac{\pi}{3}$

91. Write $\cos 195^\circ$ in terms of a reference angle $-\cos 15^\circ$

92. Write $\sin 290^\circ$ in terms of a reference angle $-\sin 70^\circ$

93. Write $\cot 140^\circ$ in terms of a reference angle $-\cot 40^\circ$

94. Given the special triangle shown with 18 inches as its longest leg length, find the lengths of the other two sides.



$18 = x\sqrt{3}$

$\frac{18}{\sqrt{3}} = x \qquad 6\sqrt{3} = x$

95. Evaluate the trigonometric expression $\sin\frac{\pi}{6} = \frac{1}{2}$

96. Evaluate the trigonometric expression $\tan\frac{\pi}{4} = 1$

97. Evaluate the trigonometric expression $\csc 150^\circ = 2$

98. Evaluate the trigonometric expression $\tan\frac{\pi}{2} =$ undefined

99. Evaluate the trigonometric expression $\sec\frac{2\pi}{3} = -2$

100. Evaluate the trigonometric expression $\cos 0 = 1$

101. Evaluate the trigonometric expression $\sin\frac{\pi}{2} = 1$

102. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

103. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) =$ $\frac{\pi}{4}$

104. $\sin^{-1}(0) =$ 0

105. $\cos^{-1}(-1) =$ π

106. $\tan^{-1}(0) =$ 0

107. $\tan^{-1}\left(\sqrt{3}\right) =$ $\frac{\pi}{3}$

108. $\sin^{-1}\left(\frac{-1}{2}\right) =$ $-\frac{\pi}{6}$

109. $\cos^{-1}\left(\frac{-1}{2}\right) =$ $\frac{2\pi}{3}$

Unit 5 is the unit we are currently studying. Please do not forget about Unit 5